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PROBLEMS OF THE WALD TEST

IN NON-LINEAR SETTINGS

AND SOME SOLUTIONS

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Problems of the Wald Test in Non-Linear Settings and Some Solutions[1]

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1. Introduction

The literature documents rich evidence of problems commonly encountered with application of the Wald test. These problems affect typically size and power properties of the test. This paper aims to provide a coherent exposition of the difficulties which cause the Wald test to be fallible using numerical examples. While discussing the problems, we illustrate as well the solutions that have thus far been suggested to improve performance of the test. This paper also puts together a list of references, admittedly not exhaustive, of some important studies discovering the anomalous behaviour of the Wald test, and those which unfold and refine the answers to the problems.

2. The Model and the Wald Test

Throughout the rest of this paper, we consider a simple linear model

$$y_t = \gamma + \beta x_t + e_t, \qquad t = 1, 2, ..., n,$$

where $e_t \sim IN(0,\sigma^2)$ and n is the sample size. The hypothesis of interest is

$$H_0: \beta - 1 = 0 \quad \text{against} \quad H_a: \beta - 1 \neq 0.$$

Following Lafontaine and White (1986), we introduce non-linearity into the testing problem by considering an algebraically equivalent formulation of the hypotheses,

$$H_0: \beta^q - 1 = 0 \quad \text{against} \quad H_a: \beta^q - 1 \neq 0, \qquad (1)$$

where q is a constant. For this model, x_t is fixed, and we use random drawings from the N(0,1) distribution as the regressor.

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Let $\theta = (\gamma, \beta, \sigma^2)'$, $h(\beta) = \beta^q - 1$, $H(\beta) = \partial h(\beta)/\partial\theta'$ and $\vartheta(\theta)$ be the information matrix. Herein after, parameters marked with ^ represent their unconstrained maximum likelihood (ML) estimators. The Wald test statistic is

$$W = h(\hat{\beta})'[H(\hat{\beta})\vartheta(\hat{\theta})^{-1}H(\hat{\beta})']^{-1}h(\hat{\beta}) \tag{2}$$

where $h(\hat{\beta}) = h(\beta)\big|_{\beta=\hat{\beta}}$, $H(\hat{\beta}) = H(\beta)\big|_{\beta=\hat{\beta}}$ and $\vartheta(\hat{\theta}) = \vartheta(\theta)\big|_{\theta=\hat{\theta}}$. The statistic W follows a chi-squared distribution with 1 degree of freedom, χ_1^2, asymptotically. Using a Taylor series expansion, it is easy to show that the asymptotic variance term for $h(\hat{\beta})$ is $V = H(\beta)\vartheta(\theta)^{-1}H(\beta)'$. Because θ is unknown in practice, it is estimated by $\hat{\theta}$, and the estimator $\hat{V} = H(\hat{\beta})\vartheta(\hat{\theta})^{-1}H(\hat{\beta})'$ is used to compute W. See Wald (1943), Stroud (1971) and Silvey (1975) for details.

For the testing problem (1),

$$V = (q\beta^{q-1})^2\,\vartheta_{22},$$

where $\vartheta_{22} = \sigma^2 / \sum_t (x_t - \bar{x})^2$ and $\bar{x} = \sum_t x / n$.

In this case,

$$\hat{V} = (q\hat{\beta}^{q-1})^2\,\hat{\vartheta}_{22},$$

where $\hat{\vartheta}_{22} = \hat{\sigma}^2 / \sum_t (x_t - \bar{x})^2$ and $\hat{\sigma}^2 = \frac{1}{n}\sum_t (y_t - \hat{\gamma} - \hat{\beta}x_t)^2$. The test statistic is

$$W = \frac{(\hat{\beta}^q - 1)^2}{\hat{V}}. \tag{3}$$

3. Size Problems

This section examines problems which can lead to serious size distortion in the Wald test. We begin by considering a data generating process (DGP) under H_0:

$$y_t = 1 + x_t + e_t, \quad e_t \sim \mathrm{N}(0, 1). \tag{4}$$

A data set with $n = 80$ was generated based on this DGP. The estimated model for this data set is

$$\hat{y}_t = 0.953 + 1.156 x_t \qquad \hat{\sigma}^2 = 1.034\,. \tag{5}$$

Using the estimates from this model, the W statistic was computed for different values of q. The results are reported in Table 1. We see that the test statistic changes in magnitude for different values of q. The p-values for these test statistics were computed based on the asymptotic distribution. It is clear that these results can be changed dramatically, from a non-rejection of the null to a strong rejection, by suitably changing the value of q. This example illustrates that the Wald test is not invariant to to different but mathematically equivalent specifications of the null hypothesis. This problem of Wald test has been examined by Gregory and Veall (1985, 1986, 1987), Lafontaine and White (1986), Breusch and Schmidt (1988) and Dagenais and Dufour (1991) in a variety of context.

Next, we study the implication of this problem on the size properties of the Wald test. 1000 samples were simulated according to the DGP defined in (4). The empirical rejection probabilities under the null were calculated by comparing the test statistic in every replication to the 5% critical value from the χ_1^2 distribution for different q values. The estimated sizes are reported in Table 2. It is clear that many of the estimated sizes differ significantly from the nominal size. The sizes vary over different q values. Increasing the sample size to 150 does not help in reducing the size distortion to a great extent. Variation in the size explains why different inferences are arrived at when the null hypothesis is reformulated equivalently. Poor size performance indicates that the χ_1^2 distribution provides a poor approximation to the distribution of the W statistic under the null hypothesis in small samples. We performed another set of experiment using $\sigma^2 = 5$. The estimated sizes differ from those for $\sigma^2 = 1$. This indicates that the Wald test is not a similar test under non-linear settings.

Given the poor size properties of Wald test, two classes of approach to find improvement in size performance of the test are common. First is to obtain a new distribution for the test statistic under the null which is closer to the true null distribution than the first-order limiting distribution. Hayakawa and Puri (1985) obtained an Edgeworth expansion to the null distribution of the Wald statistic used in the testing of simple hypotheses. Phillips and Park (1988) developed a general form for this based on an Edgeworth expansion up to order n^{-1} when non-linear restrictions are involved. The second class of approach is to find modification to the test statistic such that it is better approximated by the first-order limiting distribution. This is adopted by Ferrari and Cribari-Neto (1993) who suggested a Bartlett-type correction to the Wald statistic. In a recent Monte Carlo study, Goh and King (1996) showed that corrections to the distribution or the test statistic result in size improvement when the Wald test performs well. When the test performs poorly, the corrections yield worse results. Moreover, corrections which result in better size properties often come at the expense of loss of power.

In this light, it is clear that a better and more generic alternative is needed to find an approximation to the null distribution of Wald statistic for small-sample applications, which is unknown in practice. Given the rapid advancement in computer technology and the fast decline in computing costs, there seems to be a growing consensus that simulation methods provide this alternative. An approach which is increasingly popular in the literature is the bootstrap method. This is a resampling scheme introduced by Efron (1979), whereby information in the sample data is used repeatedly to understand a phenomenon under study, without the need to make any assumption about the DGP. An excellent survey on the method can be found in Jeong and Maddala (1993) and Horowitz (1997). The bootstrap approach has been recommended by Horowitz and Savin (1992) and Horowitz (1997) to resolve the problem of non-invariance in the Wald test.

We illustrate the application of bootstrap in our testing problem. The aim is to find an empirical distribution function for the W statistic under H_0 for the DGP in (4). We do not know the true value of γ, and its estimate of 0.953 is used. The null value of β, which is 1, is applied as we are generating a bootstrap sample under H_0. The distribution of e_t is unknown, and we draw a random sample with replacement from the demeaned residuals, $\hat{e}_t - \bar{\hat{e}}$, where $\hat{e}_t = y_t - 0.953 - 1.156x_t$, and $\bar{\hat{e}}$ is the sample mean for $\hat{e}_t$. A typical bootstrap sample is

$$\hat{\hat{y}}_t = 0.953 + x_t + \hat{\hat{e}}_t \qquad (6)$$

where $\hat{\hat{e}}_t$ is a resample from the demeaned residuals. By repeating this process for 200 times, we can compute an array of the *W* statistics, which is utilized to represent the empirical distribution function of the test statistic under H_0. From this, the 5% critical value can be estimated, and it is reported in Table 1 for different *q* values. We see that some bootstrap critical values are much larger than the asymptotic critical value of 3.841. Based on the empirical distribution functions, we interpolate the p-values, and the results, although not totally consistent, are close enough to lead us to the same conclusion at conventional significance levels for different formulation of the null hypothesis.

A further simulation study is conducted by repeating the above process for 1000 replications. The sizes were computed by comparing the test statistic obtained for every replication to the bootstrap critical value. The results are given in Table 2. None of them are significantly different from the nominal size. They are also very close to each other for different values of *q*. It is worth noting that the problem of non-similarity which we encountered earlier is resolved here. The bootstrap, although does not resolve the non-invariance problem entirely, reduces greatly the sensitivity of the test results to different formulations of the same null hypothesis such that they do not yield markedly different conclusions in practice.

4. Power Problems

We now investigate the power properties of the Wald test. Six different data sets were generated according to the DGP given by

$$y_t = 1 + \beta x_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, 1) \tag{7}$$

where β = 0.8, 0.7, 0.5, 0.2, 0.1 and 0.05, respectively. The value of *q* was set at –1. The Wald statistics are reported in Table 3 together with their p-values, the estimated distance from the null and the variance term. As we move away from the null, the *W* statistic increases at first, and we find evidence to reject the null hypothesis. When the distance from the null increases further, the *W* statistic eventually declines and H_0 cannot be rejected. This implies that the probability of rejecting the null is lower for out of the ballpark departures from H_0 than when the data is mildly inconsistent with H_0. This is known as non-monotonicity in the Wald statistic, a problem raised by Hauck and Donner (1977), Vaeth (1985) and Mantel (1987). From Table 3, we find that this problem stems from the estimated variance, $\hat{V}$, which increases at a faster rate than the estimated distance from the null, thus causing the *W* statistic to tend to zero, when the distance between the hypothesized and the true parameter values increases. Given $\hat{V} = (q\hat{\beta}^{q-1})^2 \hat{\vartheta}_{22}$, $\hat{V} \to \infty$ as $\beta \to 0$ for $q < 0$. This behaviour can be largely attributed to the use of $\hat{\beta}$ in estimating the variance term.

To circumvent this difficulty, Goh and King (1997) proposed a null Wald test for multiparameter situations by extending the principle suggested by Mantel (1987) and Laskar and King (1997) for testing problems that do not involve nuisance parameters. They recommended the use of a root-*n* consistent estimator under H_0 for θ instead of using $\hat{\theta}$ in the variance term. In our example, this estimator is $\hat{\theta}_0 = (\hat{\gamma}, \beta_0, \hat{\sigma}^2)'$ where β_0 is the null value of β. To obtain this estimator, the parameter of interest is replaced by its null value, and the nuisance parameters are replaced by their unconstrained ML estimators. In our testing problem, this estimated variance evaluated using this estimator is given by

$$\hat{V}_0 = q^2 \hat{\vartheta}_{22},$$

and the null Wald (*NW*) statistic is

$$NW = \frac{(\hat{\beta}^q - 1)^2}{\hat{V}_0}. \tag{8}$$

The *NW* statistic is distributed as χ_1^2 asymptotically. From Table 3, we see that $\hat{V}_0$ is relatively stable along the sequence of alternatives. The *NW* statistic is increasing monotonically, and evidence against the null grows stronger, with increasing distance from H_0.

By repeating this process 1000 times, we obtain the empirical power curves for the Wald and *NW* tests. The results are given in Table 4. The powers were estimated based on both asymptotic and bootstrap critical values, and for n = 80 and 150. Only the size of the *NW* test is significantly different from 5 percent in the small sample case. The use of bootstrap effectively eliminates this size distortion. On the left tail of H_0, the Wald test's power exhibits non-monotonic behaviour. However, this is not found on the right tail. Non-monotonicity in the power function of the Wald test has been reported by

Nelson and Savin (1988, 1990) and Laskar and King (1997) for different models. On the other hand, the *NW* test has a monotonic power function on both tails. The powers increase when a larger sample size is used, and this is in agreement with the notion that both tests are consistent tests.

We observe that both the tests have ill-centred power curves. This has caused the Wald power to decrease below its size at local alternatives for n = 80, and thus the Wald test is biased. Biasedness in the Wald test has been suggested by Peers (1971) and Hayakawa (1975). The *NW* test, although is monotonic in power, inherits the problem of biasedness as well. This problem, however, disappears when a larger sample size is considered. Further discussion on the problems of local biasedness of the Wald and *NW* tests is given by Goh and King (1999). To deal with this problem, they suggested a new testing procedure which uses two critical values, one to control for the size of the test, and the other to ensure local unbiasedness.

5. An Application in the Probit Model

We illustrate how the Wald test can provide misleading results due to an ill-behaved variance term in the probit model. A probit model is fitted for a subset of the data used by McManus (1985) (reprinted in Maddala (1992)). The data set consists of observations for 44 states in the United States in 1950. The dependent variable is whether capital punishment was introduced in a state. The significance of the coefficient of each of the three explanatory variables (defined in Table 5), and also the joint significance of them, were tested using the Wald, *NW* and likelihood ratio (*LR*) tests. The results are reported in Table 5. The p-values are computed from the asymptotic distribution, as well as the empirical distribution simulated using the bootstrap method with 2000 replications. The two sets of p-values are fairly close to each other. The three tests indicate consistently that the constant term, and the coefficients of X_1 and X_2 are insignificant. The Wald test suggests that the coefficient of X_3 and the joint test are insignificant, with p-values close to one. However, the *NW* and *LR* tests show that X_3 is significant at well below the 1 percent level, and the slopes are jointly significant at 5 percent. The contradicting results of the Wald test suggest the test is evaluated at a point where the power function is declining.

6. Concluding Remarks

The aim of any hypothesis test is to accurately control the probability of wrongly rejecting a true null hypothesis, while at the same time ensuring the probability of correctly rejecting a false null hypothesis is as high as possible. The advent of simulation methods lightens a burden off hypothesis testing in terms of getting the size right. However, achieving good size properties does not insure that high power will be obtained. Given the recent development in bootstrap methods, the literature is beginning to see a shift of research direction, from that aiming at developing forms of test statistics that have right sizes, toward finding forms of tests that have desirable finite-sample power properties.

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Table 1
The statistics, p-values and bootstrap critical values
for the Wald test for H_0: $\beta^q = 1$

DGP: $y_t = 1 + x_t + e_t$, $e_t \sim N(0, 1)$, $n = 80$

q	W	p-value (asymptotic)	5% bootstrap critical value	p-value (bootstrap)
1	1.261	0.262	3.499	0.263
-1	1.683	0.195	5.259	0.173
-5	3.130	0.077	13.301	0.161
-11	8.776	0.003	112.137	0.170
-23	94.665	0.000	1948.285	0.151
-25	145.580	0.000	4865.205	0.136

Table 2
The empirical sizes of the Wald test for H_0: $\beta^q = 1$

DGP: $y_t = 1 + x_t + e_t$, $e_t \sim N(0, \sigma^2)$

	Asymptotic critical value		Bootstrap critical value	
q	$\sigma^2 = 1$	$\sigma^2 = 5$	$\sigma^2 = 1$	$\sigma^2 = 5$
1	0.056	0.056	0.050	0.050
-1	0.054	0.078*	0.048	0.051
-5	0.102*	0.168*	0.049	0.049
-11	0.166*	0.251*	0.049	0.048
-23	0.247*	0.313*	0.049	0.049
-25	0.252*	0.319*	0.049	0.049
1	0.049	0.049	0.049	0.049
-1	0.049	0.073*	0.045	0.056
-5	0.090*	0.153*	0.055	0.056
-11	0.148*	0.237*	0.056	0.054
-23	0.229*	0.330*	0.053	0.054
-25	0.239*	0.342*	0.053	0.054

Notes: The results are based on a Monte Carlo simulation with 1000 replications. To compute the bootstrap critical values, 200 bootstrap iterations were used.
*Significantly different from the nominal size (5%) at the 1% significance level.

Table 3
The W and NW statistics for testing H_0: $\beta^{-1} = 1$, and the estimated distance from H_0 and two variance estimates for computing the test statistics

DGP: $y_t = 1 + \beta x_t + e_t$, $e_t \sim N(0, 1)$, $n = 80$

β	W	p-value (asymptotic)	$(\hat{\beta}^q - 1)^2$	$\hat{V}$	$\hat{V}_0$	NW	p-value (asymptotic)
0.80	1.844	0.175	0.066	0.036	0.014	4.605	0.032
0.70	3.124	0.077	0.192	0.061	0.014	13.356	0.000
0.50	4.352	0.037	1.037	0.238	0.014	72.226	0.000
0.20	1.722	0.189	16.944	9.839	0.014	1180.092	0.000
0.10	0.519	0.471	89.803	172.968	0.014	6254.388	0.000
0.05	0.131	0.717	441.050	3364.281	0.014	30717.108	0.000

Table 4
The empirical power of the Wald and NW tests for H_0: $\beta^{-1} = 1$ at 5% significance level

DGP: $y_t = 1 + \beta x_t + e_t$, $e_t \sim N(0, 1)$

β	W (asymptotic)	NW (asymptotic)	W (bootstrap)	NW (bootstrap)	W (asymptotic)	NW (asymptotic)	W (bootstrap)	NW (bootstrap)
	$n = 80$				$n = 150$			
1.0	0.054	0.077*	0.048	0.050	0.049	0.054	0.045	0.051
0.9	0.018	0.231	0.018	0.197	0.069	0.271	0.076	0.256
0.8	0.031	0.510	0.024	0.444	0.338	0.682	0.317	0.661
0.7	0.065	0.776	0.066	0.720	0.743	0.937	0.719	0.926
0.6	0.115	0.930	0.117	0.905	0.952	0.996	0.933	0.993
0.5	0.133	0.984	0.131	0.973	0.995	1.000	0.989	1.000
0.4	0.100	0.998	0.111	0.997	0.961	1.000	0.940	1.000
0.3	0.048	1.000	0.062	1.000	0.766	1.000	0.719	1.000
0.2	0.021	1.000	0.027	1.000	0.373	1.000	0.357	1.000
0.1	0.017	1.000	0.016	1.000	0.073	1.000	0.077	1.000
1.1	0.182	0.040	0.169	0.023	0.295	0.101	0.279	0.085
1.2	0.422	0.115	0.383	0.066	0.700	0.436	0.681	0.391
1.3	0.709	0.307	0.666	0.193	0.946	0.816	0.943	0.756
1.4	0.903	0.591	0.883	0.408	0.997	0.970	0.997	0.962
1.5	0.978	0.822	0.973	0.653	0.999	0.997	0.999	0.997
1.6	0.997	0.951	0.993	0.837	1.000	1.000	1.000	0.999
1.7	1.000	0.991	0.999	0.934	1.000	1.000	1.000	1.000
1.8	1.000	0.999	1.000	0.978	1.000	1.000	1.000	1.000
1.9	1.000	1.000	1.000	0.996	1.000	1.000	1.000	1.000

Notes: See footnote to Table 2.

Table 5
The probit model for determinants of propensity to have capital punishment

Explanatory variable	Coefficient	Estimated std. error	*W*	p-asym[a]	p-sim[b]	*NW*	p-asym	p-sim	*LR*	p-asym	p-sim
Constant	0.2681	0.8305	0.1043	0.7468	0.7670	0.1112	0.7387	0.7800	0.1188	0.7303	0.7644
X_1	-1.6887	1.8129	0.8676	0.3516	0.3306	0.6166	0.4323	0.4776	0.9761	0.3232	0.3361
X_2	0.0045	0.0041	1.1932	0.2747	0.2984	1.3187	0.2508	0.3187	1.3292	0.2489	0.2799
X_3	5.9798	1046.3583	0.0000	0.9954	0.9969	160.5290	0.0000	0.0000	9.9333	0.0016	0.0065
Joint significance[c]			0.0000	1.0000	0.9947	8.4935	0.0368	0.0460	10.8130	0.0128	0.0207

Notes: Dependent variable is 1 if the state has capital punishment, and 0 otherwise.
Independent variables:
X_1 = ratio of number of convictions to number of murders in 1950
X_2 = median time served in months of convicted murderers released in 1951
X_3 = 1 for southern states, 0 otherwise.
[a] p-values computed from the asymptotic distribution.
[b] p-values computed from the bootstrap simulated empirical distributions.
[c] Test for joint significance of all slopes.

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