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# Beauty and the Economist:

## The Role of Aesthetics in Economic Theory

Cassey Lee
Faculty of Economics & Administration
University of Malaya

email: cassey@um.edu.my

and

P. J. Lloyd
Department of Economics
University of Melbourne

email: pjlloyd@unimelb.edu.au

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"... but for harmony beautiful to contemplate, science would not be worth following."
Henri Poincaré

"My work always tried to unite the true with the beautiful; but when I had to choose one or the other, I usually chose the beautiful."
Hermann Weyl

"Pulchritudo splendor veritatis" ["Beauty is the splendor of Truth"]

## I. Introduction

Beauty has been discerned in scientific endeavours as well as in the arts. The importance of aesthetic considerations has been recorded in the natural sciences and in mathematics. Statements of aesthetic considerations are, however, rare in the science of economics.

Some economic theorists may consider such questions irrelevant. This view may be attributable to some features of the economics discipline or its practice. One feature is the belief of the economist that economic analysis is based on "rational/objective" considerations of agents and not on "subjective" ones such as aesthetics. This methodological view is not confined to economics only. McAllister (1996, p.7-8) calls this view the rationalist image of science. A second feature is the economist's embrace of the falsificationist methodology. This may carry with it the implication that empiricism is the final arbiter of the "truthfulness" of economic knowledge. A pejorative way of making this point would be to ask the question, "What is the use of a beautiful theory not anchored in reality?"

Philosophers of science have begun to explore the possibility that aesthetic appreciation is consistent with a rationalist methodology in science. It may motivate scientists or facilitate proofs of theorems. For economists to undertake a methodological investigation of aesthetics in the discipline is a natural step, one that acknowledges that the discipline continues to borrow extensively from mathematics and the natural sciences.[1]

This essay is not an exercise in the appreciation of beauty in economic theory or an attempt to make economic theory more beautiful. Rather, interest in this topic arises because of possible links between the aesthetics of economic theory and the way economists construct and use their theory. We seek to uncover an additional dimension of the way economists work.

In this paper, we shall argue that aesthetic considerations may play an important part in the development of economic theories. To do this, we first bring in examples of aesthetic motivations in mathematics and the natural sciences in Section II. In Section III, we investigate the recognition of aesthetics in economics. Section IV takes as a case study the pure theory of international trade and Section V examines the roles aesthetics may play in the development of economic theory.

## II. Beauty in Mathematics and the Natural Sciences

That beauty is an important consideration in mathematical discovery has been acknowledged many times in that discipline. An often-quoted statement on the topic is that of the eminent Cambridge mathematician, G.H.Hardy :

> "The mathematician's patterns, like the painter's or the poet's, must be *beautiful*; the ideas, like the colours or words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics." (Hardy, 1940, p.85).

[1] See Mirowski (1989, 1994) and Cohen (1994).

Not surprisingly, views of beauty vary among mathematicians. Some scholars have adopted the common view that beauty cannot be defined.

> "[Mathematical beauty] cannot be defined any more than beauty in art can be defined, but which people who study mathematics usually have no difficulty in appreciating it" (Chandrasekhar, 1987, p.69 quoting P. M. Dirac).

Other scholars have rejected the subjective view of beauty:

> "...given the historical period and the context, one finds substantial agreement among mathematicians as to which mathematics is to be regarded as beautiful ... the beauty of a piece of mathematics does not consist merely of the subjective feelings experienced by an observer. The beauty of a theorem is an objective property on par with its truth." (Rota, 1997, p. 126)

This rejection of the subjective view opens the way for analysis of beauty.

Some mathematicians have attempted to define beauty in mathematics. This is usually done by looking for qualities or attributes of beauty. In his *A Mathematicians's Apology*, Hardy identifies the notion of "seriousness" as essential to a beautiful mathematical theorem

In his view, two qualities which are essential to a "serious" theorem (which makes it significant) are generality and depth. On "generality", Hardy says:

> "A significant mathematical idea, a serious mathematical theorem, should be 'general' ... The idea should be one which is a constituent in many mathematical constructs, which is used in the proof of theorems of many different kinds. The theorem should be one which is capable of considerable extension and is typical of a whole new class of theorems of its kind. The relations revealed by the proof should be such as connect many different mathematical ideas." (Hardy, 1940, p. 104)

Related to "generality" is connectedness with other existing mathematical ideas. On "depth", Hardy is more opaque:

> "... *depth* ... it has *something* to do with *difficulty*; the 'deeper' ideas are usually the harder to grasp: but it is not at all the same ... It seems that mathematical ideas are arranged somehow in a strata, the ideas in each stratum being linked by a complex of relations both among themselves and with those above and below. The lower the stratum, the deeper (and in general the more difficult) the idea." (Hardy, 1940, pp. 109-110)

Another interesting quality of a beautiful mathematical theorem is "unexpectedness" or "surprise".

A more systematic analysis of mathematical beauty is given by Gian-Carlo Rota. Rota (1997, pp. 121-122) distinguishes between different "objects" in mathematics which may be beautiful. Beauty in mathematics may refer to theorems, proofs (short ones), theories (short, self-contained chapters fitting within broader theories) or definitions (new ones). When such distinctions are made, there can be cases where a given theorem is beautiful but its proof is not. In addition, Rota considers a beautiful proof to be a sufficient but not a necessary condition for a beautiful theorem:

> "A beautiful theorem may not be blessed with an equally beautiful proof; beautiful theorems with ugly proofs frequently occur ... It is however impossible to find beautiful proofs of theorems that are not beautiful."

Rota makes the important point that the assessment of mathematical beauty is dependent upon the school of thought and time to which an assessor belongs. In this light, beauty is indeed in the "eye" of the beholder:

> “ ...the beauty of a piece of mathematics is dependent upon schools and periods. A theorem that is in one context thought to be beautiful may in a different context appear trivial.” (Rota, 1997, p. 126)

The time or school-dependence of mathematical beauty can be attributable to the requisite investment in the mathematical skills:

> “Familiarity with a huge amount of background material is the condition for understanding mathematics. A proof is viewed as beautiful only after one is made aware of previous clumsier proofs.” (Rota, 1997, p. 129-130)

Perhaps, such training prepares the ground for ‘instantaneous’ appreciation of beauty in mathematics.

> “...mathematical beauty should be appreciated with the instantaneousness of a light bulb being lit.” (Rota, 1997, p.130)

This is related to the notion of “surprise” that Hardy wrote about. One must possess intuitive familiarity (and thus some expectations) before one can be caught by surprise. In addition, such investments in mathematical skills are essential to the appreciation of ‘deep’ results to which Hardy alludes.

Finally, Rota is also very explicit about the role of ‘connectedness’ in mathematical beauty:

> “The beauty of a theorem is best observed when the theorem is presented as the crown jewel within the context of a theory ... a theorem is beautiful when they mean to say that the theorem is enlightening. We acknowledge a theorem’s beauty when we see how the theorem ‘fits’ in its place, how it shed light around itself, like *Lichtung*, a clearing in the woods.” (Rota, 1997, pp. 130-132)

For a sample of beautiful objects in mathematics see Huntley (1970), King (1992), Lang (1985), Rothstein (1995) and Chageux and Connes (1995).

Like their counterparts in mathematics, some natural scientists have been frank about aesthetics in their discipline. These include Poincaré, Dirac, Einstein, Heisenberg, and Feynman. The list would be significantly larger if we include scientists' work as evidence in itself as well as self-comments on their aesthetic motivations. We would then include giants such as Copernicus, Galileo, and Newton.

For some, beauty has been considered no less than the primary motivation for doing science:

> "The scientist does not study nature because it is useful to do so. He studies it because he takes pleasure in it; and he takes pleasure in it because it is beautiful. If nature were not beautiful, it would not be worth knowing and life would not be worth living ... I mean the intimate beauty which comes from the harmonious order of its parts and which a pure intelligence can grasp." (Henri Poincaré quoted by Chandrasekhar, 1987, p.59.)

An important issue at stake in analyzing the importance of aesthetic considerations in the natural sciences is the relationship between 'truth' and beauty. Werner Heisenberg refers to the tendency to associate the two notions in the following quote:

> "If nature leads us to mathematical forms of great simplicity and beauty - by forms I am referring to coherent systems of hypotheses, axioms etc. - forms that no one has previously encountered, we cannot help thinking that they are 'true', that they reveal a genuine feature of nature ... You must have felt this too: the almost frightening simplicity and wholeness of the relationships which nature suddenly spreads out before us and for which none of us was in the least prepared." (quoted by Chandrasekhar, p.65)

Such views raise difficult questions. In particular, aesthetic considerations in the natural sciences may differ from those in mathematics. In so far as beauty relates to "truth", the reality in which "truth" applies in mathematics is an abstract reality defined within the system by the axioms employed. The natural sciences, in contrast, have to deal with a reality distinct from the formal mechanisms it employs, i.e. it is subject to empirical verification. In commenting on natural phenomena, the mathematician Hardy (1940, p. 47) put it this way:

"It cannot be possible to prove mathematically that there will be an eclipse tomorrow, because eclipses, and other physical phenomena, do not form part of the abstract world of mathematics." The relation between truth and beauty, in this context, takes on a different meaning.

Furthermore, because the natural sciences are embedded in a physical reality, a natural scientist may resort to the use of visualization-related techniques (such as thought experiments) in the formulation of theories.[2] While sense-based visual imagery is a limited technique, this can be overcome by the use of mathematics. In some cases, one may even argue that the use of mathematics helps to extend the boundaries of visualization by providing new modes of visual representation. Indeed, visualization may be subsumed by mathematical abstraction altogether. Still, some scientists continue to have preference for visualizing theories (McAllister, 1996, p.50). In the end, both have a place in science.

In cases where mathematics is used extensively, beauty in the natural sciences would be partly derived from mathematical aspects of mathematically formulated natural science theories. If this is the case, we can say that this beauty comes from the language (i.e. mathematics) used.

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[2] See Kuhn (1977), Brown (1991), Miller (1996), and McAllister (1996, pp.48-54)

Views regarding the close association between truth and beauty have received circumstantial confirmation from cases in which beautiful theories turn out to be true even if, at the time of formulation, it appeared to be otherwise. An example is Weyl's two-component relativistic wave equation of the neutrino (Chandrasekhar, p.66). (See also Feyman and Gell-Mann's experience reported in Miller, p.393.) In the more extreme case, empirical verification may not even matter, i.e. beauty takes precedence over empirical evidence! The physicist P. M. Dirac, for example, relied on aesthetic criteria in assessing theories:

> "It is more important to have beauty in one's equations than to have them fit experiments ... It seems that if one is working from the point of view of getting beauty in one's equations, and if one has really a sound insight, one is on a sure line of progress." (Chandrasekhar, 1987, p. 66)

Similarly,

> "The foundations of the theory (of general relativity) are, I believe, stronger than what one could get simply from the support of experimental evidence. The real foundations come from the great beauty of the theory ... It is essentially beauty of the theory which I feel is the real reason for believing in it." (McAllister, 1996, p.16).

As in discussions of aesthetics in mathematics, natural scientists have considered the qualities for beauty. Chandrasekhar (1987, p. 70) has listed two "criteria" for beauty in science and applied them carefully to the specific example of Einstein's Theory of Relativity. The first has to do with "strangeness", "wonderment" and "surprise". In this respect, he quotes Francis Bacon: "There is no excellent beauty than hath not some strangeness in the proportion." This criterion is an affective attribute which can provide useful clues as to whether a theory is beautiful or not. The second criterion is that of connectedness to other theories. Such connectedness may sometime take the form of analogies between different domains (McAllister, 1996, pp.44-48). It is interesting to note that both analogical as well as metaphorical interpretations of a theory may be sources of aesthetic pleasure, the latter more in the arts.[3]

A related notion is that of universality ( unity, unified theories ) (Miller, 1996), p.393). Interestingly, both of these qualities/criteria have also been cited by mathematicians. To have characteristics such as connectedness and universality would seem to imply a third - consistency (with existing theories/structures).

Other aesthetic qualities include that of Hutcheson's "uniformity amidst variety" and "symmetry". McAllister (1996, pp.41-44) has a discussion of symmetry in relation to a few "beautiful" theories in the physical sciences. However, as noted above, aesthetic criteria may change over time in the natural sciences. An example in physics is the acceptance circa 1957 of asymmetry as a characteristic law of nature. (See Miller, 1996, p.385-386) and the fascinating book by Weyl (1952).

For a sample of theories, theorems, proofs and equations that have been recognised as beautiful in the natural sciences, see McAllister (1996), Chandrasekhar (1987, chapter 4) and Farmelo (2002).

**III. Beauty in Economic Theory**

Having presented evidence that colleagues in other disciplines are influenced by aesthetic considerations in their work, we consider now whether economists are similarly affected. Direct evidence is sparse as very few economists have openly acknowledged aesthetic considerations as being important in their research. There is no collection of the views of economists about beauty in economic theory to our knowledge.

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[3] McAllister (1996, p.48). Use of metaphors and analogies may be regarded as rhetorical devices by some. Such a view would fall neatly into McCloskey's "economics as the art of persuasion" interpretation.

We begin with a catalogue of some instances in economic theory where beauty has been recognised and acknowledged. One of the most explicit statements about the possible existence of beautiful objects in economics comes from Paul Samuelson:

> "There is a viewpoint, which I used to share, that the mathematics of pure mathematics and of physics is intrinsically beautiful, but the mathematics of economics is of an inferior aesthetic order. I think this view is wrong. The simple Ricardian theory of comparative advantage is beautiful, and let no mathematician persuade you otherwise." (Samuelson, 1954, p.382).

General equilibrium theory is an area where the beauty of the theory has been extolled. Shackle (1966, p. 64) saw beauty in the general equilibrium itself:

> "If we ask at what epoch economic theory attained its greatest beauty, there can be no doubt of the answer. Its age of Periclean glory lies in the last decade of the nineteenth century and the first of the twentieth… At the end of them, Pareto's work was the full and final blossoming of the great tree planted by Walras…the Great Theory, the demonstration that a single principle (the self-interested seeking of maximum results from given expenditure whether by producers or consumers) could explain the whole structure and metabolism of economic life, by applying to the factor market that marginal analysis which had in the 'seventies been applied to the product market. The theory had a wholeness, an inclusive unity and a severity and simplicity of logic which are scarcely matched in science save by classical astronomy and by the table of atomic weights in chemistry."

Hahn (1985, pp. 20-21) declared

> "…Who can read the Core Theorem without even a flicker of pleasure in the beautiful and not obvious role of the Arrow –Debreu equilibrium?"

One obvious and notable feature of these instances is that all are made by eminent economists who, presumably, are sufficiently well respected in the discipline to risk making such admissions. The "elite" includes Paul Samuelson, Kenneth Arrow, and Gerard Debreu, all Nobel laureates (the ultimate accolade in the discipline). This is not to say that only modern economists are aware of the importance of aesthetic considerations. Mark Blaug (1992, p.53), for example, hinted at similar views being held by a much-revered predecessor:

> "[Adam] Smith credited the origin of science in the essay on astronomy, not to men's idle curiosity or the impulse to master nature, but to the simple desire to assuage "wonder, surprise, and admiration". Even his standard of judgment of scientific ideas was more often aesthetic than strictly cognitive ... "

There have been few explorations of the nature and role of beauty in economics. Unlike mathematics or the natural sciences, there have been no books written on beauty in our discipline, nor even any essays in the style of Hardy (1940) and Rota (1997, chapter X) for mathematics or Chandrasekhar (1987, chapter 4) for the natural sciences.

In so far as economic theory is mathematically formulated, beauty may derive from the nature of the mathematics used. That economic theorists themselves have discerned the beauty of mathematics is a recorded fact. Debreu and Kenneth Arrow admit the influence of mathematical aesthetics in their work:

> "... the austere beauty of mathematics ... A seductive subject presented by a dedicated master at a critical time in my intellectual development pointed me, unaware, toward a career of scientific research."(Debreu, 1992, p.110)

"Mathematics is certainly a source of aesthetic pleasure. Over and over we have the sense of symmetry, of elegance, of an abstract and pervasive unity of seemingly disparate parts. My mathematical skills and taste for abstraction led me to emphasise the aesthetic aspect of mathematics. As with many others, this aesthetic bent has found its place in my scholarly work." (Arrow, 1992, p.49)

These views are relevant to the motivation behind the mathematization of economic theory. Debreu elaborates on this

"Mathematics provides him with a language and a method that permits an effective study of economic systems of forbidding complexity; but it is a demanding master. It ceaselessly asks for weaker assumptions, for stronger conclusions, for greater simplicity... mathematics also dictates the imperative of simplicity. It relentlessly searches for short transparent proofs and for the theoretical frameworks in which they will be inserted. Participating in that pursuit, economic theory was sometimes drawn by drives towards greater generality and towards greater simplicity in the same direction, rather than in opposite directions." (Debreu, 1992, p.4)

But what of economic theories which are not formalised mathematically? Suppose we apply Marshall's decree that the use of mathematics be minimised. Could we still detect beauty in such theories? The answer must be yes. Beauty is not confined to mathematics.

To progress beyond casual observations, we need to consider the nature and role of beauty in economic theory. This in turn requires that we define the objects of beauty and the qualities or attributes that may be a part of the recognition of beauty in objects of economic theory as a specific area of scientific creativity.

To define the objects of beauty, we use a list of classes of objects. Our list is

- Definitions
- Theorems
- Proofs
- Equations
- Models
- Theories

This list is a modification of the list of Rota (1997). Theories in this context are a set of models with a high degree of commonality in subject matter or methods of analysis: for example, neoclassical theory or general equilibrium theory.

Two classes of objects have been added to the list provided by Rota (1997). The first is "models". Economic theory has progressed to construct models that are tailored to explain phenomena in the real world and which therefore have variables, identities and behavioural relations to suit particular circumstances in the real world. Essentially, these circumstances relate to particular markets, or sets of markets such as in the specification of a general equilibrium model of some country. In this respect, economic theory resembles the theories of the natural sciences rather than the purely abstract theory of mathematics.

The second class of objects added is "equations". Several examples of equations in the natural sciences that are regarded as beautiful are discussed in Farmelo (2001). (See also the example in Chandresakhr, 1987, p. 61). Equations include functions that play a prominent part in the maximising behaviour of economic agents. For convenience, we shall use the term 'economic theory' hereafter to cover all six classes of objects.

One can conceive of beauty in any of the classes of objects in economic theory. The examples given above all related to theories or theorems. One can conceive of a beautiful proof in economics, as in mathematics; for example the one-line proof of the Slutsky Equation by Cook (1972) scores very highly on the attributes of simplicity and surprise.

One can conceive of a beautiful equation or function in economics though we do not know of any which have been described in these terms.

This list of classes of objects is not exhaustive. One could add, for example, stories in the sense of McCloskey (1983), that is, an "extended example of the reasoning underlying the mathematics".

One work (an article or book) may contain more than one object and possibly objects from two or more different classes. Each object contained within the work may be considered beautiful or not. For example, we may consider the beauty of a theorem and/or of its proof. It is possible for a beautiful theorem to have an ugly (non-beautiful) proof and logically for the converse too. Rota (1997) gives examples from mathematics of beautiful theorems with ugly proofs but claims that there are no beautiful proofs of ugly theorems.

Discussions of beauty in mathematics and natural sciences proceed as if each beholder of an object has an index or a function that measures the magnitude of beauty. These functions are individual-specific. They have multiple arguments, which we shall call attributes. It seems reasonable to suppose there is a similarity in preferences to the extent that the functions of all economists will have a common set of arguments and they will agree on the measurement of each attribute.[4] Thus, the functions provide a mapping from multi-dimensional attribute space to the real line.[5] That is, for the individual observer h,

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[4] The most general form of the function is

$$B^h = f^h(A^h_{1_h}, \ldots, A^h_{n_h})$$

where $(A^h_{1_h}, \ldots, A^h_{n_h})$ is the set of $n_h$ attributes for individual h and $f^h$ is the function that maps these attributes from $n_h$-space to the real line.

Using this function, one can interpret the statement that "beauty lies in the eye of the beholder" as reflecting three distinct sources of individual perception of beauty. The first is that the list of attributes may be individual-specific. The second is that measurement of each attribute may be individual-specific. The third is that the function mapping these attributes is individual-specific. The first two forms of individual specificity have been ruled out in Equation (1).

[5] An interesting attempt to quantify beauty was made by the mathematician Birkhoff (1933). He constructed an "aesthetic measure" and calculated its numerical value for a variety of objects; musical harmonies, polygons and other geometric shapes, poems, vases and ornaments. His measure is M = O/C where O is the "order" and C is the "complexity" of the object. O and C in turn are written as linear functions of variables,

$$B^h = f^h(A_1, \ldots, A_n) \qquad (1)$$

$(A_1, \ldots, A_n)$ is the set of n attributes. Little can be said about the properties of the functions except that they are non-decreasing in all arguments because attributes are defined as positive qualities. Are they continuous, or are they just a binary classification as the use of the terms "beautiful" and "ugly " suggest?[6] Are they bounded?[7] Are they concave or convex? There seems little reason to presume that any of these properties must hold. We can, however, consider the attributes.

For a list of possible attributes that comprise beauty, we draw upon those suggested for mathematics and the natural sciences and upon some of the distinctive features of economic theory. The list of possibilities includes

- generality
- simplicity
- symmetry
- depth
- surprise
- realism

These attributes are a mixture of form and content. Simplicity was remarked upon in the quotation above by Debreu. Symmetry has been advanced as a possible attribute in mathematics. These two aspects relate to form. They are reminiscent of an often-quoted definition of beauty by Heisenberg:

> "beauty is the proper conformity of the parts to one another and to the whole."

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which include vertical and rotational symmetry. Hence $M \in (0, +\infty)$ but, for some classes of objects, he narrows the limits. In his view, the aesthetic measure is not individual-specific.

[6] Rota (1997, p. 131) asserts "Mathematical beauty and mathematical truth share one important property. Neither of them admits degrees." This is questionable in the case of mathematical beauty.

[7] This is equivalent to the question – is there perfection in beauty or can any level of beauty be surpassed?

‘Surprise’ is a matter of content rather than form. In particular, it relates the content of a theorem or model or another object relative to what has gone before it in the literature of economic theory. It has also been remarked upon in both mathematics and the natural sciences. “Generality” in this context bears on the relationship between, say, a theorem and the theorems and propositions which have preceded it and which may follow it. As noted above, this gives an element of time and school-dependence to the appreciation of beauty.

Realism is a feature of an applied science like economics. It may apply to all of the classes of objects except proofs. In commenting upon the allure of mathematics in economics, Samuelson (1954, p. 382) added:

> “But let me not be misunderstood. Part of the extra beauty of any applied mathematics lies in its applicability to some reality. Extra zest comes from following the rules of the game; and it is part of the rules of the game of economic theory that your deductive creations be of empirical relevance.”

In this respect, economics is like the natural sciences and unlike mathematics.

In one other respect, economics is different than the natural sciences.

> “I know of no analogs in engineering, or even in physics, to questions pertaining to the desirability of perfect competition, or to that of ‘free’ international trade, or even of markets. Unlike engineering, questions of desirability do not reduce solely to considerations of ‘prediction and control’ .” (Khan, 1993, p. 797-98)

This refers to the normative branch of economics in the customary divide between positive and normative economics. Or, as Varian (1989, p. 10) expressed it, “…economics is fundamentally a policy science”. The normative role of economic theory and the emphasis on optimisation may affect our perceptions of beauty in the objects of economic theory.

Some attributes may not be relevant to some classes of objects. For example, generality and realism do not apply to proofs.

**IV The Pure Theory of International Trade as a Case Study**

It may be advantageous to take an area of theory within the discipline of economics as a case study and investigate it in some depth. The pure theory of international trade is a subset of general equilibrium theory, one of the areas of economics where beauty has been remarked upon. It is the set of general equilibrium models of the world economy, where the world comprises a number of countries that trade goods and sometimes (non-produced) factors among each other. It contains a number of well-known and important theorems. These include the theorem, the so-called Principle of Comparative Advantage, which has been called "[the] deepest and most beautiful result in all of economics" (Findlay, 1987, p. 514).
To make the investigation manageable, we focus on the Heckscher-Ohlin theory of international trade with brief comment on the earlier Classical model and on one post-Heckscher-Ohlin model.

The most famous theorem in the pure theory of international trade is the Principle of Comparative Advantage. The theorem was originally stated by Ricardo (1817) in the context of what is now called the Classical model of international trade.

<u>The Principle of Comparative Advantage</u>

*Each country exports the good that it produces relatively cheaply before international trade.*

This theorem has been heralded as beautiful. Its beauty lies partly in its form and partly in its content. The form derives from the 2x2 structure of the model; there are two countries and two goods. The "evenness" of the dimensions of the model gives it symmetry. One compares the relative price of the two goods between the two countries. Thus, there is a double relativity in the theorem. One country must have a comparative advantage in one good and a comparative disadvantage in the other good and, conversely, the other country must have a comparative disadvantage in the first good and a comparative advantage in the second. The content of the theorem has generality. The result modifies the earlier Principle of Absolute Advantage and the principle holds equally for the Heckscher-Ohlin and the Jones specific-factor models and most (but not all) other post-Classical models of international trade. In the Heckscher-Ohlin and later models, the source of comparative advantage of nations may lie in international differences in resource endowments as well as in differences of the technology of production as in the Classical model. The content of the theorem is "surprising" or counter-intuitive too. It is significant or 'serious' in the sense of Hardy. The principle shows that the nations gain from trade in the sense that the aggregate consumption sets of the countries are expanded, even though the trade is undertaken by private agents acting only in their own interests to determine a competitive equilibrium for the world economy.[8] It provides the intellectual justification for a huge and rapidly increasing amount of human activity, namely, cross-border transactions.

In the terminology of our classes of objects, the Heckscher-Ohlin theory is a model. The model was developed by Heckscher (1919) and Ohlin (1933). It is a Neoclassical model with the usual Neoclassical assumptions of perfect information, complete markets, constant returns to scale in all production activities and perfect competition in all markets. Perfect competition in all markets implies that there is no strategic interdependence between agents, as there is in some post-Heckscher-Ohlin models. We shall confine our investigation to the 2x2x2 version as it is known; the version of the model with two countries, two produced goods and two non-produced factors. This is the version used in the original statement of all theorems examined below except the factor price equalisation theorem.

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[8] This interpretation of the gains from trade is due essentially to Samuelson but is consistent with that of

There are four basic theorems derived from the model:

- The Heckscher-Ohlin Theorem
- The Factor Price Equalisation Theorem
- The Stolper-Samuelson Theorem
- The Rybczynski Theorem

One feature of the theorems stands out in their historical context. The statement of three of the theorems has changed substantially since the first statement by the original authors. Only one – the Rybczynski Theorem - was stated in the form that is now standard and even now, several decades after the initial statement of each, there is some variation in the statements of each theorem. The form used in the statement of the theorems below is a standard one similar to that found in contemporary textbooks.

<u>The Heckscher-Ohlin Theorem</u>

Version I (The weak version)

*Each country exports the good that uses intensively the factor that is relatively cheap before international trade.*

Version II (The strong version)

*Each country exports the good that uses intensively the factor with which the country is relatively well endowed.*

This theorem explains the pattern of specialisation laid down in the Principle of Comparative Advantage in terms of the relative abundance of the two factors in the two countries. The distinction between the two versions was first drawn clearly by Jones (1956): the first defines abundance in terms of relative factor prices and the second in terms of relative physical endowments. Both versions require assumptions additional to those usually imposed on the model, including the assumption of the identity of the technologies (=industry production functions) across the two countries. Interestingly, a

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Ricardo.

complete proof of the strong version was not provided until Riezman (1974). Earlier proofs were incomplete. No claims have been made for the beauty of this theorem, perhaps because it explains the rather mundane matter of which country exports which good.

<u>The Factor Price Equalisation Theorem</u>

*Goods price equalisation leads to factor price equalisation.*

This theorem was enunciated and proven by Samuelson (1949, 1950). His proof requires a number of assumptions additional to those usually imposed on the model. These include the absence of barriers to trade in goods that leads to the equalisation across countries of the prices of all goods, incomplete specialisation (both countries produce both goods in strictly positive quantities), the identity of the technologies across the two countries and restrictions on the technology such that the production functions are homogeneous of degree +1 and each good uses one factor more intensively than the other for all factor prices (the "Samuelson strong factor intensity assumption").

The theorem provides a set of conditions that are sufficient for factor price equalisation. An alternative set of sufficient conditions was provided by Chipman (1966). The necessary and sufficient conditions were provided decades later by Blackorby, Schworm and Venables (1993).

No claims have been made for the beauty of this theorem. It has the form property of surprise. Ohlin had asserted that equalisation of goods prices produced a "…tendency towards equalisation of the prices of factors of production" (Ohlin, 1933, p. 49). It is certainly deep as it stems from the interdependence of all prices in a competitive equilibrium and involves global properties of the model that determine the pattern of specialisation. However, it is not general as it does not apply to a number of models of comparative advantage such as the Classical model, the Jones factor-specific model and the Heckscher-Ohlin model itself with systematic differences in technology among the countries.

## The Stolper-Samuelson Theorem

*An increase in the relative price of a good will increase the real incomes of the factor that is used intensively in the production of the good, and decrease the real income of the other factor.*

The Stolper-Samuelson Theorem (and the Rybczinski Theorem) relate to behaviour in only one of the trading economies, taking the prices of the goods traded as given. Stolper and Samuelson (1941) stated a theorem that predicted the movement of the real incomes of factors in an internationally trading economy that may protect an industry and, in effect, owners of certain factors employed in the industry. At the heart of this effect is a simple and strong relationship between goods prices and factor prices "that has nothing to do with factor scarcity or abundance and is independent of whether prices change because of protection or for any other reason." (Deardorff, 1994, p. 12). Deardorff called this the "essential" version and it is the version now used by economists. In the introduction to the volume celebrating the Golden Jubilee of the Stolper-Samuelson Theorem, Deardorff (1994) provides a detailed history and anatomy of the various versions of the theorem.

This version has some appeal aesthetically. In the 2x2 version (two factors and two goods, dropping the third pair as the theorem relates to only one economy) of the model, it has the symmetry of the Principle of Comparative Advantage and the Heckscher-Ohlin Theorem. It was surprising as it contradicted the beliefs among economists at the time that no factor would lose from trade liberalisation once factor proportions had adjusted. But its main appeal lies in the content. The theorem is immensely important. Its importance derives from its key message: goods price changes necessarily create conflict between households owning different factors. Subsequent extensions of the theorem are the foundation of political economy models of tariffs, other taxes and government interventions.

The Rybczynski Theorem

*An increase in the endowment of one factor will, at constant prices, increase the output of the good that uses that factor intensively, and reduce the output of the other good.*

The investigation of the theorem put forward by Rybczynski (1955) is the most straightforward of the four. The statement of this theorem has not changed, though later researchers note that the proportionate increase in the one output is greater than the proportion by which the endowment increases. Unlike the other comparative static theorem, the Stolper-Samuelson theorem, there has been little interest in extending the factor intensity explanation of the result to higher dimensional models. The Rybczynski Theorem is used in the proof of the Heckscher-Ohlin Theorem which is now standard (see Riezman, 1974)

The proofs of all the three theorems other than the Rybczynski theorem have been the subject of a large literature. (Chipman, 1965 a and b and1966 provides a masterly survey of the development of trade theory up to the mid-Sixties.) This was partly because of some imprecision or inadequacy in the original proofs and differences between local and global versions of the theorems. It is also partly due to the vexatious and deep problem of dimensionality as the results do not extent in the original form to dimensions higher than 2x2x2 though all theorems can be extended to dimensions with any fixed number of countries, goods and factors (lxmxn) with an appropriate redefinition of terms. These problems make them unlikely candidates for beautiful proofs.

We could consider the beauty of the Heckscher-Ohlin model itself as an object. The theorems are all deep as they stem from the interdependence of all endogenous variables in a competitive equilibrium of the model. They are intimately related to each other.

Amazingly, the Stolper-Samuelson Theorem and the Rybczynski Theorem turn out to be the same theorem in a mathematical sense. The two theorems had been put forward independently. Both are theorems concerning the comparative static properties of the model. One is a relationship between an (exogenous) change in the price of some good and the induced change in the prices of factors, and the other a relationship between an (exogenous) change in the quantity of some endowed factor and the induced change in the outputs of the industries. In each case, the direction and magnitude of the changes are explained in terms of factor intensity conditions.

The formal identity of the essential results predicting the directions of change was discovered by Samuelson (1953, p.10). Oddly, this recognition by Samuelson was published in 1953 before the publication of the Rybczynski Theorem itself in 1955, but Samuelson did not state the factor intensity condition of the Rybczynski result. Samuelson's proof of the identity of the essential results used the fact that the derivatives showing the relationship between a change in the price of a good and the change in the price of a factor, and the change in the output of a good and the change in the endowment of a factor, are each the second derivatives of another function, the national product function. The result follows immediately from Young's Theorem, which states that higher order derivatives of a function are independent of the order of differentiation, applied to the second derivatives of the function.

The proof of this Samuelson theorem is itself a candidate for a beautiful proof in economic theory. It uses the symmetry of Young's Theorem. And to do so, Samuelson had to invent a completely novel functional relationship, the national product function, which has shown to be of great value subsequently in general equilibrium analysis. This theorem as an object also ranks very highly in terms of surprise. The two prior results relate to different comparative static propositions involving completely different variables. The new result brings out a very deep duality relationship that springs from the interdependence of all endogenous variables in a general equilibrium model, one which readers of the separate theorems had not suspected but recognised instantaneously. However, the result is purely mathematical, no one has yet provided any intuition for it. It is quite general as it applies to a variety of models of production with constant returns to scale. Hence, we may have an example in the theory of international trade of an "ugly" theorem with a "beautiful" proof.

The last model from the pure theory of international trade that we consider is that of Krugman (1979, 1980). This was a model that departed radically from the pre-existing Heckscher-Ohlin model in several respects. It introduced strictly increasing returns to scale at the level of the firm in all production activities; to overcome the problems of producer interdependence and strategic behaviour, it adopted the Chamberlinian device of a large number of firms in an industry; there is only one industry, that which produces a set of differentiated products that are close (but imperfect) substitutes for each other; there is only one factor, labour; and the number of goods produced and consumed are endogenously determined.

Given these changes, the properties of the model differ radically from those of the Heckscher-Ohlin and other models. To begin with, there is no comparative advantage in the model. Moreover, which countries produce which goods is indeterminate, though the number of goods produced and consumed is determinate. There are gains from trade but they are due to a cause missing from previous models, the "love of variety". Contrary to the Heckscher-Ohlin model, all households gain from trade. Factor prices are, however, equalised.

This model is a strong candidate for a beautiful model. In form, it has an enormous amount of symmetry in both production and consumption and great simplicity in terms of having only one industry and one factor. In content, it had great surprise. Some of the most able international economists had endeavoured to construct and solve models with increasing returns to scale at the firm level but none had succeeded. (There were, however, models with economies of scale external to the firm.) There was a widespread belief that it could not be done, with, in particular, much concern over the non-existence and multiplicity of equilibria (see Chipman, 1965b, pp. 739-749). It had considerable generality.  Krugman's model was a major breakthrough in the history of the construction of trade models.  It precipitated a revolution in trade theory, leading to a number of other new models of trade.  These became known as "new trade theory".  These models incorporated a more complex specification of the structure of production and competition within multiple-output industries and emphasised intra-industry trade.

## V The Role of Beauty

The discussion of Sections III and IV revealed relatively few instances of the recognition and acknowledgment of beauty in economic theory compared to mathematics or the natural sciences.  Consequently, what needs principally to be explained in economic theory is not the nature of beauty, where it exists, but the poverty of instances in which beauty has been recognised and acknowledged.

In the tradition of economics, we adopt a utilitarian approach to analysis of the role of beauty.  Beauty must perform some useful role in the construction or use of economic theories or it must be an incidental by-product.  What are the roles that beauty might play in the development of economic theory?

These roles may be put in the form of questions

- Does beauty provide motivation?
- Does beauty aid discovery?
- Does beauty aid acceptance or adoption of theory?
- Does beauty aid empirical verification?

The role in the fourth dot point cannot have played a role in many cases because typically the persons who develop a theory and those who test them are disjoint.  Hence, our focus will be on the others.

Motivation has been posed in discussions of aesthetics in mathematics in particular. Most economists, we expect, would subscribe to the view that rewards other than aesthetics drive economic innovators. Could it be the economic motive itself – real income? While there are rewards for research productivity, the payoffs are extremely uncertain and long lagged. If not fortune, is it fame?

> "Scientists are as avaricious and competitive as Smithian businessman. The coin they seek is not apples, nuts, and yachts: nor is it coin itself, or power as that term is ordinarily used. Scholars seek fame. The fame they seek…is fame with their peers - the other scientists whom they respect and whose respect they strive for." (Samuelson, 1985, p. 72).

Similarly, in commenting on his career, Krugman (2002, p. 1) states

> "I may have been in pursuit of Truth and Beauty, but I, like everyone, was also in pursuit of success."

Or could it be, since economics is a policy science, a desire to improve the world, that is, the economy in which we live? Again we have no data on this aspect of the production of knowledge. We note the view of Keynes:

> "Marshall always used to insist that it was through ethics he arrived at political economy and I would claim myself in this, as in other respects, to be a pupil of his. I should have thought that nearly all English economists in the tradition, apart from Ricardo, reached economics in this way. There are practically no issues of policy as distinct from technique which do not involve ethical considerations." (quoted in O'Donnell, 111989, p. 165).

Nor does beauty, where it exists, seem to aid discovery in economic analysis. If the scant examples discussed above are representative, beauty in economic theory is found predominantly in models and theorems and perhaps in theories, not in proofs, definitions or equations. The motivation for most modellers is to explore the behaviour of economic agents and to use this knowledge to manage the micro- or macro-economy to the betterment of the citizens of the country or countries concerned. This absence of a role for beauty in the creation of economic theory may explain the paucity of acknowledgments of beauty in economic theory.[9]

This still leaves a possible role as an aid to the acceptance or adoption of a theory, once discovered. The perception that an object is beautiful may aid its becoming known and its popularity.[10] Indeed, this may be the most important role of beauty in economic theory. One possible reason why particular models become adopted widely in textbooks and other sources may be that they are tractable and can be represented conveniently by graphs or equation systems. In some cases, one version of a model may triumph for the same reasons. One illustration of this may be Hicks's representation of the Keynesian system by the IS/LM graph. (On this aspect of the adoption of Keynes's General Theory, see Young, 1987). Another example may be the Phillips curve representation of the trade-off between inflation and unemployment. Simplicity combined with seriousness and without a great sacrifice of generality may be the key to adoption success.

The success of the Krugman model may illustrate a related effect of beauty, namely, seduction. Although a number of models with multiple-output industries and intra-industry trade followed Krugman in the 1980s, his model was adopted by many as the explanation of observed intra-industry trade with no empirical evidence to support this adoption. Indeed, empirical studies of intra-industry trade have now shown that most of the product differentiation associated with observed intra-industry trade is vertical differentiation associated with differences in the qualities of good rather than the horizontal differentiation of the Krugman love-of-variety type. In reviewing this empirical literature, Davis and Weinstein (2001, pp. 9-10) observe

[9] There is another logical possibility. Economists may be reluctant to admit beauty where it has been recognised. There is, however, no reason to expect reticence on the part of economists.

[10] "… a beautiful argument will be imitated…" (Rota, 1997, p. 126).

> "In the end, the Krugman-Lancaster approach to intra-industry trade became the prevailing view because it could be presented in an elegant, comprehensive, and compelling framework that tied together what were viewed as key stylized facts."

There may be other cases of the seduction of economic modellers in the history of economic analysis.[11] Keynes's well-known comments on the controversy between Malthus and Ricardo over 'effective demand' may be interpreted in a similar way:

> "one cannot rise from a perusal of this correspondence without a feeling that the almost total obliteration of Malthus' line of approach and the complete domination of Ricardo's for a period of a hundred years has been a disaster to the progress of economics." (Keynes, 1933, p. 98).

His explanation is most interesting in the present context: "… it was Ricardo's more fascinating intellectual construction which was victorious …" (Keynes, 1933, p. 87)[12]

In conclusion, the quest for beauty in economic theory has not been highly rewarding. The quest is severely handicapped by any data regarding economists' views on beauty. There are few instances in which beauty has been acknowledged in economic theory.

Nonetheless, this quest does raise a number of important and generally neglected issues concerning the factors that stimulate the production and adoption of innovations in economic theory. In the analysis above, one could substitute for "beauty" some other attribute such as "cleverness", "novelty" or "importance" and then seek to explain how economic theory advances in this direction. Little research has been done on any of these determinants of the advancement of economic theory.

[11] Our attention was drawn to this case by Robert Dixon.

[12] Keynes also discusses this example in *The General Theory*: "The completeness of the Ricardian victory is something of curiosity and a mystery…". One of the factors he mentions is "… that it was adapted to carry a vast and consistent logical superstructure, gave it beauty." (Keynes, 1936, p. 33).

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