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SECTORAL RESPONSES TO EXOGENOUS DISPLACEMENTS: A SUGGESTED MODE OF ANALYSIS FOR A BIMODULAR SYSTEM*

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ABSTRACT

The study is built on the premise that a macroeconomic model and an input-output model represent coexistent and different facets of the same equilibrium which makes them fundamentally compatible. If we treat the array of input-output coefficients as proxies to first-order derivatives, these can be recomputed about their respective means to yield proportional changes. Similarly, the macrosystem can generate a matrix of elasticities relating exogenous changes to endogenous reactions in the sense of Johansen-Tower. We suggest a mathematical framework whereby these elements are conjoined to map the effects of changes in percentage terms beginning with exogenous sources that work through macrovariables and ultimately displace sectoral production levels. Variations in demands placed on sectoral output owing to any one exogenous perturbation are the end result of both macroeconomic sensitivity and differences in industrial linkage. The mechanism is innovative in the sense that it proposes elasticities rather than absolute changes be taken about a multi-dimensional economic equilibrium point.

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1. Introduction

As economies become increasingly and strategically intertwined no country remains impervious to shocks emanating from another part of the globe, particularly the relatively open and smaller economies. As economies exhibit varying degrees of interdependence and external vulnerability, the aftermath felt by countries following any global shift will differ in terms of strength and importance.

While the impact of international linkages are important for the economy as a whole, another issue of concern in any economy is the difference in impact across sectors within the country, given their diverse economic strengths and weaknesses. This further suggests dissimilarities in terms of speeds of adaptation, sensitivity and types of solution to a given situation. A large number of policy issues and decisions are undertaken at the macroeconomic level and may comprise sub-optimal solutions for separate industries. To say that 'one size fits all' is overly simplifying. There appears to be a need for tailor-made policies applicable to individual sectors of growth. The problem has a dual thrust, one based on the macro level outlook and another from the sectoral standpoint.

One of the ways to address highly varied yet connected and dependent issues is to take stock of inter-industrial connectivity in addition to aggregate macro-economic analyses thus adopting a holistic analytical approach. Responses to an external factor will induce repercussions throughout the economy and any attempt of focus on only one or two aspects would be tantamount to ignoring the multidimensional nature of the development problem.

As an economy becomes a more complex entity, studies aimed at simulating its structure must accommodate this need.

2. A Solution: The Proposed Bimodular Framework

One solution lies in an integrated approach through a bimodular study based on the linkage of a macro-model with an input-output system. We attempt to mathematically construct the linkages using a series of elasticities, the product-matrix of which may be used to explain net regional output changes owing to external shifts. The macromodel yields the first matrix of elasticities explaining final demand component changes following exogenous shifts. As detailed in Yap (1998) the relevant elasticities are taken from the Johansen-Tower (Johansen (1960), Tower (1984), Tower and Loo (1989)) inverse which means all related n endogenous variables in the macrosystem can be accommodated, either directly or indirectly. The second and third matrices are drawn from the input-output model. The second matrix comprises elasticities linking initial demands on sectoral output with given adjustments in final demand components. The third matrix holds elasticities which explain net sectoral output changes in response to displacements in sectoral final demand.

The rationale for using the chosen two models is explained at this juncture. A dynamic macromodel can serve as a valuable analytical tool by endogenising a large number of variables and therefore explaining the relationships between them. In our study, it provides the framework for the evaluation of macroeconomic sensitivity to external shocks. It thereby addresses the need for a system which can depict the present day economic make-up within an increasingly open environment.

However, a macromodel on its own does not account for interindustrial relations and their link with demand components which drive them. While the aggregate components generated by any macromodel may provide an indication of overall trends in the economy, they are inadequate for industrial analysis or planning where finer supply-side breakdowns may be necessary. For example, an investment shock and its demands on output are separated by multiple transactions involving the exchange of services and goods amongst economic units. The macromodel on its own does not explain the exact manner in which production units respond and interact, the proportion and nature of adjustment or subsequent sectoral multiplier effects.

The issue of planning involves as much the analysis of gross requirements as it does net demands. The various rounds of output generation requiring input acquisitions preceded by necessary levels of production and so forth comprises the essence of an input-output approach. Such an approach can discern and capture the intermediate steps which separate initial demand signals to producers and their final delivery for end consumption.

If demand and supply factors react according to some given pattern of response and if production technology and trade flows can be used to determine the link between output and input, then a connective fabric of responses would provide the basis for a detailed study on economic interactions.

By linking an input-output system with a macromodel, income and expenditure in the input-output framework can be tied to other relevant economic factors either via direct or indirect pathways. This paper will be devoted to a theoretical discussion on the mechanism of our proposed pathway linking a series of changes.

The main differences between this form of bimodular articulation (to use the terminology of Jin and Wilson (1993) and other known linkages (as in Hewing (1985) or Jin and Wilson (1993)) are as follows. For one, the framework here works through a system of first order derivatives taken about their respective means. This reduces to a series of elasticities which, in turn, hinge upon the parameters derived for each module. Most known linkages on the other hand, work through levels or values of macro and regional variables.

Secondly, other linkages largely involve exogenous change which displace demand levels which determine regional supply. Percent changes have to be computed about their starting values in a second step. Therefore, their empirical application necessitates the determination of a new set of endogenous values for each and every exogenous-variable combination. Net changes which result have to be repeatedly computed as a percentage shift from initial values in subsequent analysis.

In comparison, our suggested link-up provides a means to directly ascertain elasticities of supply variables with respect to exogenous factors without the need for a two-stage procedure. This is because it deals exclusively with percent changes as opposed to variables. It maps changes from their external source right down to sectoral supply responses whereby intermediate adjustments in expenditure (and other macro-variables) have been explicitly accounted for.

3. **Motivation**

Our primary impetus comes from the observation that the effect of exogenous perturbations on industrial activities has not been directly defined owing to the structured layout of the input-output system. Most applications of input-output modules have kept to standard procedures whereby the supply-driven or demand-driven mechanism is adopted, analysis is based on variable levels and linkages are dependent upon what comprises inputs and outputs to component modules (termed the connective approach by Jin and Wilson (1993)).

Previous integration studies function on series of variables defined in; *in level terms, not derivatives*; thus they produce solution vectors of values as opposed to rates of change pertaining to endogenous factors. Our suggested mode of linkage therefore offers another way of looking at connected changes particularly in systems comprising two or more modules.

The other method involves embedding an input-output module (another term put forward by Jin and Wilson (1993)) whereby a bimodular framework uses one calculation system.

The method we propose more closely resembles the embedding mode although the theoretical sense of it is quite distinct from other approaches. We postulate that the component modules are representative of alternative facets of one equilibrium thus ensuring compatibility. This traces an alternative analytical path towards a desired goal.

4. **The Theoretical Premise and Discussion**

Charting the outcome involves three matrices. The first matrix is drawn from the macromodel and the second and third matrices derived from the input-output model, hence the term ‘bimodular integration’.

This paper focuses on the integrated system of analysis. Derivation of the individual matrices may be found in Yap (1998) and will be discussed in a series of forthcoming papers.

The procedure rests on the following assumptions:

- Firstly, that the macroeconomic system may be represented by the macromodel and that the interindustrial system is approximated by the input-output model.

- Secondly, that both systems can be linearised about the initial solution.

- Thirdly, that both systems represent different facets of the same equilibrium and are therefore compatible.

- Fourthly, that net changes in one variable relative to change in another given by coefficients in the input-output inverse can be seen as alternative proxies for first-order derivatives.

- Fifth, that these derivatives taken about their respective means yields ratios which read as supply elasticities.

The rules that govern composite functions are first presented. We then go on to explore the possibility that composite functions can be reproduced as elements in a product matrix using appropriately processed matrices. It may be argued that the fundamental or underlying characteristics of the two systems lend themselves to an approach which uses a series of either derivatives or elasticities to map exogenous changes down to sectoral responses.

Suppose we have a function,

$$z = f(y) \qquad (1)$$

where y is a function of another variable x,

$$y = g(x) \qquad (2)$$

then the derivative of z with respect to x is equal to the derivative of z with respect to y times the derivative of y with respect to x. Thus:

$$\frac{dz}{dx} = \frac{dz}{dy} \frac{dy}{dx} = f'(y)g'(x) \qquad (3)$$

We may also express the function

$$z = f(y)$$

as $z = f\ [g(x)]$ (4)

where the contiguous presence of 2 functions amounts to a composite function i.e. a function of a function (Chiang 1984).

This chain rule can be extended to three or more functions. If z = f (y), y = g (x) and x= h (w) the derivative of z with respect to w is given by

$$\frac{dz}{dw} = \frac{dz}{dy}\ \frac{dy}{dx}\ \frac{dx}{dw} = f'(y)\ g'(x)\ h'(w) \qquad (5)$$

A similar rule applies to elasticities

$$\varepsilon_{yw} = \varepsilon_{yx}\ \varepsilon_{xw} \tag{6}$$

Where the proof is as follows:

$$\begin{aligned} \varepsilon_{yx}\varepsilon_{xw} &= \left(\frac{dy}{dx}\ \frac{x}{y}\right)\left(\frac{dx}{dw}\ \frac{w}{x}\right) \\ &= \frac{dy}{dx}\ \frac{dx}{dw}\ \frac{x}{y}\ \frac{w}{x} \\ &= \frac{dy}{dw}\ \frac{w}{y} \\ &= \varepsilon_{yw} \end{aligned} \tag{7}$$

Coefficients in the input-output inverse are regarded as solutions for unknown I's (total output) in terms of given F's (final output). This can be presented as:

$$I_1 = b_{11}\,F_1 + b_{12}\,F_2 + \ldots + b_{1n}\ F_n \tag{8}$$

$$I_n = b_{n1}\,F_1 + b_{n2}\,F_2 + \ldots + b_{nn}\,F_n$$

Total differentiation for each equation yields:

$$dI_i = b_{i1}\,dF_1 + b_{i2}\,dF_2 + \ldots + b_{in}\,dF_n \quad where\ i=1,\ldots,n \tag{9}$$

If changes only occur in the F_j^{th} component then all other terms drop out of the expression. This leaves us with:

$$dI_i = b_{ij}\ dF_j \tag{10}$$

$$\therefore \frac{dI_i}{dF_j} = b_{ij} \tag{11}$$

In other words each coefficient b_{ij} can be looked upon as a partial derivative describing changes in production of i subject to changes in F_j when all other things remain equal. The point elasticity of I_i with respect to F_j is defined as:

$$\varepsilon_{Iifj} = \frac{dI_i / dF_j}{I_i / F_j} = \frac{\textit{marginal function}}{\textit{average function}} \tag{12}$$

Each derivative taken about its mean provides us with an elasticity estimate. It is suggested that the ratio of total sectoral output for the i^{th} industry I_i to the total final output for the j^{th} industry F_j may provide an approximation for the average function we seek. The value generated as:

$$\varepsilon_{IiFj} = \frac{dI_i \ / \ dF_j}{I_i \ / \ F_j} \tag{13}$$

is assumed equivalent to the supply elasticity of commodity i relative to final demand for commodity j.

In a similar vein, coefficients for final output, say ξ_{jk}, provide the proportion of final output j contained in any one unit of final demand Y_k. For example, ξ_{jk} = 0.05 would imply that for every dollar increase in the Y_k^{th} demand component, the effective increase in demand for the j^{th} commodity would be 0.05 or 5 sen. It follows that the total amount of final output j required to meet demand amounting to Y_k would be:

$$F_{jk} = \xi_{jk} Y_k \tag{14}$$

Differentiation yields:

$$dF_{jk} = \xi_{jk}\ dY_k \tag{15}$$

$$\frac{dF_{jk}}{dY_k} = \xi_{jk} \qquad j = 1, \ldots n \qquad\qquad k = 1, 2, \ldots K \tag{16}$$

As Y_k refers to final demand components then k = 1 applies to private consumption, k = 2 to public consumption and so forth. We are not concerned with sum totals here and the omission of stocks will not influence the outcome on elasticities obtained. Its average function will be given by the ratio of total final output F_j to total final demand Y_k. Percent deviations in final output j subject to a one percent change in final demand type k will be proxied by:

$$\varepsilon_{F_j Y_k} = \frac{dF_{jk} / dY_k}{F_j / Y_k} \tag{17}$$

The policy matrix derived from the macromodel gives the outcome on any endogenous variable in terms of percent deviations from the initial solution set subject to unit percent deviations in an exogenous variable. Production activities in the input-output module are driven by demand components whose responses to external factors can be explained by the macro-module. Therein lies the basis of our linkage. We therefore require a submatrix of elasticities pertaining to consumption (private and government), investment and exports drawn from the main matrix. These show displacements in demand components following shifts in given exogenous components. Reactions by all other endogenous variables are implicitly incorporated into these elasticities which represent net effects. Each elasticity is given by:

$$\varepsilon_{Y_k X_\ell} = \frac{dY_k / dX\ell}{Y_k / X\ell} \qquad (18)$$

which relates the percent response in a given demand variable to a one percent change in the exogenous factor.

Matrices are dealt with as shown in Figure 1. The first matrix comprises percent changes in industrial output with respect to final output changes, the second matrix holds percent changes in final output with respect to final demand changes and the third matrix contains percent changes in final demand with respect to exogenous variable changes. Dimensions for the first matrix are (n by n), for the second matrix (n by K) and for the third matrix (K by m). This results in a product matrix which should conform to dimensions of n by m.

Suppose we illustrate by focusing on one final demand component, such as private consumption. The intermediate product matrix will be derived as shown in Figure 2.

Figure 1: The Proposed Conjoined System

$$
\begin{bmatrix}
\frac{dI_1/I_1}{dF_1/F_1} & \frac{dI_1/I_1}{dF_2/F_2} & \cdots & \frac{dI_1/I_1}{dF_n/F_n} \\
\frac{dI_2/I_2}{dF_1/F_1} & \cdots & \vdots & \cdots \\
 & \cdots & \frac{dI_i/I_i}{dF_j/F_j} & \cdots \\
 & & \vdots & \\
\frac{dI_n/I_n}{dF_1/F_1} & \frac{dI_n/I_n}{dF_2/F_2} & & \frac{dI_n/I_n}{dF_n/F_n}
\end{bmatrix}
\begin{bmatrix}
\frac{dF_1/F_1}{dCP/CP} & \frac{dF_1/F_1}{dCG/CG} & \cdots & \frac{dF_1/F_1}{dI/I} & \frac{dF_1/F_1}{dEx/Ex} \\
\frac{dF_2/F_2}{dCP/CP} & \frac{dF_2/F_2}{dCG/CG} & \cdots & \frac{dF_2/F_2}{dI/I} & \frac{dF_2/F_2}{dEx/Ex} \\
\vdots & \vdots & & \vdots & \vdots \\
\frac{dF_j/F_j}{dCP/CP} & \frac{dF_j/F_j}{dCG/CG} & & \frac{dF_j/F_j}{dI/I} & \frac{dF_j/F_j}{dEx/Ex} \\
\frac{dF_n/F_n}{dCP/CP} & \frac{dF_n/F_n}{dCG/CG} & & \frac{dF_n/F_n}{dI/I} & \frac{dF_n/F_n}{dEx/Ex}
\end{bmatrix}
\begin{bmatrix}
\frac{dCP/CP}{dX_1/X_1} & \cdots & \frac{dCP/CP}{dX_\ell/X_\ell} & \cdots & \frac{dCP/CP}{dX_m/X_m} \\
\frac{dCG/CG}{dX_1/X_1} & \cdots & & \cdots & \frac{dCG/CG}{dX_m/X_m} \\
\frac{dI/I}{dX_1/X_1} & & & &
\end{bmatrix}
$$

Figure 2 : First Matrix Operation:

An Illustration Using One Final Demand Component Only.

$$
\begin{bmatrix}
\frac{dI_1/I_1}{dF_1/F_1} & \frac{dI_1/I_1}{dF_2/F_2} & \cdots & & \cdots & \frac{dI_1/I_1}{dF_n/F_n} \\
\frac{dI_2/I_2}{dF_1/F_1} & \frac{dI_2/I_2}{dF_2/F_2} & \cdots & & \cdots & \frac{dI_2/I_2}{dF_n/F_n} \\
\vdots & \vdots & & \vdots & & \vdots \\
 & \cdots & \cdots & \frac{dI_i/I_i}{dF_j/F_j} & \cdots & \vdots \\
\vdots & & & \vdots & & \vdots \\
\frac{dI_n/I_n}{dF_1/F_1} & \cdots & \cdots & \cdots & \cdots & \frac{dI_n/I_n}{dF_n/F_n}
\end{bmatrix}
\begin{bmatrix}
\frac{dF_1/F_1}{dCP/CP} \\
\frac{dF_2/F_2}{dCP/CP} \\
\vdots \\
\frac{dF_j/F_j}{dCP/CP} \\
\vdots \\
\frac{dF_n/F_n}{dCP/P}
\end{bmatrix}
$$

(n x n) (n x 1)

=

$$
\begin{bmatrix}
\frac{dI_1/I_1}{dF_1/F_1} \bullet \frac{dF_1/F_1}{dCP/CP} + \frac{dI_1/I_1}{dF_2/F_2} \bullet \frac{dF_2/F_2}{dCP/CP} + \cdots + \frac{dI_1/I_1}{dF_n/F_n} \bullet \frac{dF_n/F_n}{dCP/CP} \\
\vdots \quad \vdots \quad \vdots \\
\vdots \quad \vdots \quad \vdots \\
\vdots \quad \vdots \quad \vdots \\
\frac{dI_n/I_n}{dF_1/F_1} \bullet \frac{dF_1/F_1}{dCP/CP} + \cdots \cdots + \frac{dI_n/I_n}{dF_n/F_n} \bullet \frac{dF_n/F_n}{dCP/CP}
\end{bmatrix}
$$

(n x 1)

$$
= \begin{bmatrix} \sum_{j=1}^{n} & \dfrac{dI_1 / I_1}{dF_j / F_j} \bullet \dfrac{dF_j / F_j}{dCP / CP} \\ \vdots & \\ \sum_{j=1}^{n} & \dfrac{dI_n / I_n}{dF_j / F_j} \bullet \dfrac{dF_j / F_j}{dCP / CP} \end{bmatrix}
$$

(n x 1)

The summations we see can be explained as follows. Imagine a one percent increase in private consumption. The change in demand can be met only if final output of every industry changes in accordance with given elasticities. Each final output change in turn will draw on output from every industry through its direct and indirect requirements. For example, the first expression consists of a sum of n terms each of which is a product of two elasticities. The first product in this summation gives us the percentage change in the output of commodity 1 subject to a one percent change in private consumption which works through increments in final output of commodity 1. The second term provides us the percentage change in output of commodity 1 subject to a one percent increase in private consumption transmitted through alterations in final output of commodity 2 and so forth. It is postulated that these responses can be summed because they represent percentage change in the same industry with respect to the same stimulus (household spending). Therefore, if industry 1 must raise output by a certain proportion for every final output reaction towards private consumption change, then the total increase in output of commodity 1 is equivalent to its sum of changes. Each sum reduces to one term which can be interpreted as the total change in its corresponding industrial output which accommodates a one percent increase in private consumption. As such, every industry effects a response equivalent to its given sum. If we limit our analysis to private consumption-induced output changes then the intermediate product-matrix generated stands as a (n by 1) column of percent changes.

The next step involves mapping demand changes caused by external impulses down to induced output adjustments. The link-factor(s) common to both inter-industrial and macro-economic systems is the proportional change in one or more demand variables. In our example using a single demand component, matrices are multiplied as shown in Figure 3.

Figure 3: Second Matrix Operation:

An Illustration Using One Final Demand Component Only.

$$\begin{bmatrix} \dfrac{dI_1 / I_1}{dCP / CP} \\ \dfrac{dI_2 / I_2}{dCP / CP} \\ \vdots \\ \dfrac{dI_i / I_i}{dCP / CP} \\ \vdots \\ \dfrac{dI_n / I_n}{dCP / CP} \end{bmatrix} \begin{bmatrix} \dfrac{dCP / CP}{dX_1 / X_1} & \dfrac{dCP / CP}{dX_2 / X_2} & \cdots & \dfrac{dCP / CP}{dX_\ell / X_\ell} & \cdots & \dfrac{dCP / CP}{dX_m / X_m} \end{bmatrix}$$

(n x 1) (1 x m)

resulting in a matrix of dimensions (n x m) :

$$= \begin{bmatrix} \dfrac{dI_1/I_1}{dCP/CP} \bullet \dfrac{dCP/CP}{dX_1/X_1} & \dfrac{dI_1/I_1}{dCP/CP} \bullet \dfrac{dCP/CP}{dX_2/X_2} & \cdots & \dfrac{dI_1/I_1}{dCP/CP} \bullet \dfrac{dCP/CP}{dX_\ell/X_\ell} & \cdots & \dfrac{dI_1/I_1}{dCP/CP} \bullet \dfrac{dCP/CP}{dX_m/X_m} \\ \dfrac{dI_2/I_2}{dCP/CP} \bullet \dfrac{dCP/CP}{dX_1/X_1} & \dfrac{dI_2/I_2}{dCP/CP} \bullet \dfrac{dCP/CP}{dX_2/X_2} & \cdots & \dfrac{dI_2/I_2}{dCP/CP} \bullet \dfrac{dCP/CP}{dX_\ell/X_\ell} & \cdots & \dfrac{dI_2/I_2}{dCP/CP} \bullet \dfrac{dCP/CP}{dX_m/X_m} \\ \vdots & & & \vdots & & \vdots \\ \dfrac{dI_n/I_n}{dCP/CP} \bullet \dfrac{dCP/CP}{dX_1/X_1} & & \cdots & \dfrac{dI_n/I_n}{dCP/CP} \bullet \dfrac{dCP/CP}{dX_\ell/X_\ell} & \cdots & \dfrac{dI_n/I_n}{dCP/CP} \bullet \dfrac{dCP/CP}{dX_m/X_m} \\ & & (n x m) & & & \end{bmatrix}$$

Each elasticity produced $\varepsilon_{\ell i X_\ell}$ tell us the percentage adjustment in output which industry i must make subject to a one percent change in the exogenous variable ℓ and given the relationships of both variables with household spending. Elasticities across a row apply to changes in the same industry in response to different exogenous perturbations from 1 to m. Elasticities down a column show percent displacements in all industrial output from 1 to n in response to the same exogenous change.

The system can be similarly expanded to include all final demand components. The intermediate matrix would then comprise of as many columns as there are categories of final demand. The intermediate product matrix is post-multiplied by another with as many rows. A generalised representation would be as shown in Figure 4 whereby CP (household consumption), CG (government consumption) etc, are replaced by Y_1 (final demand component 1), Y_2 (final demand component 2) and so forth. The Second Matrix Operation in Figure 4, illustrates the multiplication process whereby all final demand components are accommodated.

Figure 4: Second Matrix Operation:

A Generalised Representation for K final Demand Components

$$\begin{bmatrix} \frac{dI_1/I_1}{dY_1/Y_1} & \frac{dI_1/I_1}{dY_2/Y_2} & \cdots & \frac{dI_1/I_1}{dY_k/Y_k} \\ \frac{dI_2/I_2}{dY_1/Y_1} & \frac{dI_2/I_2}{dY_2/Y_2} & \cdots & \frac{dI_2/I_2}{dY_k/Y_k} \\ \vdots & & & \\ \frac{dI_n/I_n}{dY_1/Y_1} & \frac{dI_n/I_n}{dY_2/Y_2} & \cdots & \frac{dI_n/I_n}{dY_k/Y_k} \end{bmatrix} \begin{bmatrix} \frac{dY_1/Y_1}{dX_1/X_1} & \cdots & \frac{dY_1/Y_1}{dX_m/X_m} \\ \frac{dY_2/dY_2}{dX_1/X_1} & & \frac{dY_2/Y_2}{dX_m/X_m} \\ \vdots & & \vdots \\ \frac{dY_k/Y_k}{dX_1/X_1} & \cdots & \frac{dY_k/Y_k}{dX_m/X_m} \end{bmatrix}$$

(n x K) (K x m)

$$= \begin{bmatrix} \sum_{k=1}^{K}\left(\frac{dI_1/I_1}{dY_k/Y_k}\bullet\frac{dY_k/Y_k}{dX_1/X_1}\right) & \sum_{k=1}^{K}\left(\frac{dI_1/I_1}{dY_k/Y_k}\bullet\frac{dY_k/Y_k}{dX_2/X_2}\right) & \cdots & \sum_{k=1}^{K}\left(\frac{dI_1/I_1}{dY_k/Y_k}\bullet\frac{dY_k/Y_k}{dX_m/X_m}\right) \\ \sum_{k=1}^{K}\left(\frac{dI_2/I_2}{dY_k/Y_k}\bullet\frac{dY_k/Y_k}{dX_1/X_1}\right) & \cdots & \cdots & \sum_{k=1}^{K}\left(\frac{dI_2/I_2}{dY_k/Y_k}\bullet\frac{dY_k/Y_k}{dX_m/X_m}\right) \\ \vdots & & & \vdots \\ \sum_{k=1}^{K}\left(\frac{dI_n/I_n}{dY_k/Y_k}\bullet\frac{dY_k/Y_k}{dX_1/X_1}\right) & \cdots & \cdots & \sum_{k=1}^{K}\left(\frac{dI_n/I_n}{dY_k/Y_k}\bullet\frac{dY_k/Y_k}{dX_m/X_m}\right) \end{bmatrix}$$

(n x m)

$$\begin{bmatrix} \varepsilon_{I1X1} & \varepsilon_{I1X2} & \cdots & & \cdots & \varepsilon_{I1Xm} \\ \varepsilon_{I2X1} & \varepsilon_{I2X2} & & \vdots & & \varepsilon_{I2Xm} \\ \vdots & & & \varepsilon_{IiX\ell} & & \vdots \\ \vdots & & & \vdots & & \vdots \\ \varepsilon_{InX1} & \varepsilon_{InX2} & \cdots & & \cdots & \varepsilon_{InXm} \end{bmatrix}$$

(n x m)

Although Figures 2 to 3 illustrate the use of two separate matrix multiplications, the actual procedure just involves two consecutive multiplications.

Inclusion of several final demand components in the analysis will not alter dimensions of the product-matrix which remain at (n by m). However, every element in this matrix will now constitute a sum of K terms where K equals the number of demand components studied. Suppose four classes of demand were incorporated namely household spending, government consumption, gross fixed capital formation and exports. Then the first term in each sum traces the effects of an exogenous impulse on output from the relevant industry which works through the private consumption component, the second through public consumption, the third through investment and the fourth through exports. Therefore, every term in one summation evaluates the outcome of an identical exogenous disturbance on output within the same industry; the only difference between terms lie in their intermediary demand component. The sum of percentage increases provide us with the total output adjustment in a given industry i likely to follow a one percent change in an external perturbation type ℓ. When empirically worked out, each summation reduces to a single expression which can be likened to an elasticity measuring proportional change in one variable (industrial output) subject to proportional change in another variable (the exogenous factor).

The technique rests on the assumptions of both models used as well as their structural build-up. We are suggesting that if A influences B on the one hand and if B affects C on the other, then some pathway probably exists which links A to C. By keeping to proportional changes the problem of differing units does not arise. Using what are conceptually composite functions, a framework is given to work out connected reactions starting from a given stimulus. If C is some function of B which is some function of A then C can be given by a reduced form expression in A. The technique however circumvents the need for explicitly stated reduced forms. More important are net changes which can be approximated by a chain product of proportional changes. The theoretical linkup is proposed as an alternative way of integrating systems which are essentially components of the same economic paradigm.

5. Conclusion

The paper's primary thrust is the design of a connective framework based on an alternative perspective in modular integration. This uses a system of products of first order derivatives taken about their respective means which interpret transmitted charges initiated by disturbances to the general equilibrium.

The study explores another avenue of research into frameworks comprising integrated multimodels. It demonstrates, using a bimodel, that a problem may be apprehended from various angles and that integration draws out the more pertinent features of each model. It highlights system compatibility and articulation to facilitate the investigation of a variety of issues, stressing more comprehensive methods of analysis.

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